Disturbance Observer Based Sliding Mode Control For Flexible Joint Manipulator Systems

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ABSTRACT
This paper presents a external disturbance observer (DO) for flexible joint single link manipulator. Besides, the arbitrary small attraction region is obtained by using the suitable parameters. The main result of this paper is proposed based on differential equation analysis. Moreover, several exploration in depending of parameters are given out. Simulation results pointed out the good behaviour of the proposed methods.

CCS Concept:
Computing methodologies → Systems theory

Keywords
Disturbance Observer (DO), Sliding Mode Control (SMC), Single link manipulator, flexible joint.

1. INTRODUCTION
Almost industrial systems are affected by external disturbances, such as manipulator control systems, robotic systems, etc. The Flexible Joint is a phenomenon including a loose connection between the motor and the link in the installation, or because of the material's substantial twisting properties, which results in a different angle of rotation of the motor and the arm. Soft couplings are an important accessory in pipe design, which connects the parts together to ensure the stability of the operating system. In addition, the coupling also functions to significantly reduce the load, while preventing overload. Many control scheme have been utilized based on robust adaptive control without estimating disturbance [5-9]. Beside removing disturbances that probably can not measuring, we estimate them to design sliding mode control based on disturbance observer (DO) [2]. However, authors in [2] have not discussed about the arbitrary small attraction region by using the suitable parameters. In [3], the proposed controller ensures the improvement of disturbance attenuation. In contrast, the reduction of computation amount is the important task. Besides, external disturbance mentioned depend only on time. Our work uses the sliding mode control to absolutely obtain arbitrary small attraction region based on the suitable parameters. Moreover, we explorer the influences of parameters to obtained results. The paper is organized as follows: The second section, we focus on problem statements. Next section, we pay attention to design the proposed controller for single link manipulator and explorer the influences of parameters. The fifth section, we present the simulation results providing evidences for these analysis. Final section concludes the brief.

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2. PROBLEM STATEMENTS
We consider the flexible joint manipulator being described by the following equation [2]:

$$\begin{cases} D(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + g(q_1) = K(q_2 - q_1) \\ J\ddot{q}_2 + K(q_2 - q_1) = u \end{cases} \quad (1)$$

Where q_1 , q_2 (n x 1) are the angles of motor, link, respectively. J is the inertia moment matrix of n links. K is the stiffness matrix of joint. D and C is the square matrice n x n.

However, if we consider the system in general form, the setting the standard form of the DO is difficult. Because if the extra subspace are $q_{1i}, \dot{q}_{1i}, q_{2i}, \dot{q}_{2i}$ with i is selected from 1 to n, then we only return to the system of 2n variables and have 2n equations. So that we obtain n equation containing the control input u.

We continue to consider the single link manipulator.

Consider a single link manipulator is described as follows:

$$\begin{cases} I\ddot{q}_1 + MgL\sin(q_1) + \mathrm{K}(q_1 - q_2) = 0 \\ J\ddot{q}_2 + K(q_2 - q_1) = u \end{cases} \quad (2)$$

Where I, J are the inertia moments of joint, motor, respectively. M and L are mass and length of link. K is the stiffness of joint.

The control objective is to find the control torque of each joint to obtain that the angle converges to attraction region in presence of external disturbances

3. EXTERNAL DO BASED CONTROL

Initially, we concentrate on designing the DO as described:

$$\begin{cases} \dot{x}_1 = \dot{x}_2 + d_1(x,t) \\ \dot{x}_2 = \dot{x}_3 + d_2(x,t) \\ . \\ . \\ . \\ \dot{x}_{n-1} = x_n + d_{n-1}(x,t) \\ \dot{x}_n = a(x) + b(x)u + d_n(x,u,t) \\ y = x_1 \end{cases} \quad (2)$$

where $x = [x_1\ x_2\ ...\ x_n]^T \in \mathbb{R}^n$ is the vector of state variables, $u \in \mathbb{R}$ is the control input and $y \in \mathbb{R}$ is the output signal. The external disturbance $d_i(x,t)\ i = 1,2,...,n-1$ does not depend on control input, while the disturbance $d_n(x,u,t)$ depends on $u \in \mathbb{R}$. $a(x), b(x)$ are continuous functions.

Assumption 1: Every disturbance is continuous and satisfy the inequalities:

$$\left| \frac{d^j d_i(x,t)}{dt^j} \right| \le \mu\ i = 1,2,...,n;\ j = 0,1,...,r$$

The system (1) is affected by disturbances $d_i(x,t)\ i = 1,2,...,n-1$, we implement the observer to estimate disturbances based on the following formulas as described in [2]:

$$\begin{cases} \hat{d}_i^{(j-i)} = p_{ij} + l_{ij}x_i \qquad i = 1,2,...,n-1 \\ \dot{p}_{ij} = -l_{ij}(x_{i+1} + \hat{d}_i) + \hat{d}_i^{(j)} \quad j = 1,2,...,r-1 \\ \dot{p}_{ir} = -l_{ir}(x_{i+1} + \hat{d}_i) \end{cases} \quad (3)$$

On the other hand, towards the component $d_n(x,u,t)$, the corresponding observer has been pointed out [2]:

$$\begin{cases} \hat{d}_n^{(j-1)} = p_{nj} + l_{nj}x_n \\ \hat{p}_{nj} = -l_{nj}(a(x) + b(x)u + \hat{d}_n) + \hat{d}_n^{(j)} \quad j = 1,2,...,(r-1) \\ \dot{p}_{nr} = -l_{nr}(a(x) + b(x)u + \hat{d}_n) \end{cases}$$

(4)

where p_{ij} are auxiliary variables and l_{ij} are arbitrary positive constants ensuring that
$$D_i = \begin{bmatrix} -l_{i1} & 1 & 0 & ... & 0 \\ -l_{i2} & 0 & 1 & ... & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ -l_{i(r-1)} & 0 & 0 & ... & 1 \\ -l_{ir} & 0 & 0 & ... & 0 \end{bmatrix}$$
have all eigenvalues belonging to the left side of complex coordinate plane.

In [2], the sliding surface has been selected

$$\sigma = \sum_{i=1}^{n} c_i x_i + \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} c_{j+1}\hat{d}_i^{(j-i)}; c_n = 1$$

and the modified sliding surface: $\sigma^* = \sigma - \sigma(0)e^{-\alpha t}$

where α is arbitrary positive constant.

The control input has been proposed in [2]:

$$u = -\frac{1}{b(x)}\left[a(x) + \hat{d}_n + \alpha\sigma(0)e^{-\alpha t} + \sum_{i=1}^{n} c_i(x_{i+1} + \hat{d}_i) + k_l\sigma^* + k_s sat\sigma^*\right] \quad (5)$$

$k_l, k_s > 0$

Where: $sat\sigma^* = \begin{cases} \operatorname{sgn}\sigma^*\ \ khi\ \ |\sigma^*| > \varepsilon \\ \dfrac{\sigma^*}{\varepsilon} \quad khi\ \ |\sigma^*| \le \varepsilon \end{cases}$

The stability analysis has been proved in [2].

Remark 1: We absolutely adjust the speed of observer error by selecting the eigenvalues.

We give transformation from (1) into (2) by using:

$$\begin{cases} z_1 = q_1 \\ z_2 = \dot{q}_1 \\ z_3 = q_2 \\ z_4 = \dot{q}_2 \end{cases} \quad (6)$$

We obtain:
$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -\dfrac{MgL}{I}\sin(z_1) + \dfrac{K}{I}(z_3 - z_1) \\ \dot{z}_3 = z_4 \\ \dot{z}_4 = \dfrac{K}{J}(z_1 - z_3) + \dfrac{u}{J} \end{cases} \quad (7)$$

In order to obtain (2), we define:

$$(x_1 \quad x_2 \quad x_3 \quad x_4)^T = (z_1 \quad z_2 \quad \frac{Kz_3}{I} \quad \frac{Kz_4}{I})^T$$

We get the following equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 - \dfrac{MgL}{I}\sin(x_1) - \dfrac{K}{I}x_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{K}{J}x_1 - \dfrac{I}{J}x_3 + \dfrac{u}{J} \end{cases} \quad (8)$$

However, because the manipulator is affected by external disturbance and uncertainties, therefore we obtain new dynamic equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 - \dfrac{MgL}{I}\sin(x_1) - \dfrac{K}{I}x_1 + \Delta\dfrac{MgL}{I}\sin(x_1) + \Delta\dfrac{K}{I}x_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_3 = \dfrac{K}{J}x_1 - \dfrac{I}{J}x_3 + \dfrac{u}{J} + \Delta\dfrac{K}{J}x_1 + \Delta\dfrac{I}{J}x_3 + \Delta\dfrac{u}{J} + \dfrac{v}{J} \end{cases} \tag{9}$$

where $-0.1 \le \Delta \le 0.1$ is described by parameter error, v is the external disturbance of input. Using

$$\begin{cases} d_2(x) = -\dfrac{MgL}{I}\sin(x_1) - \dfrac{K}{I}x_1 + \Delta\dfrac{MgL}{I}\sin(x_1) + \Delta\dfrac{K}{I}x_1 \\ d_4(x,t) = \Delta\dfrac{K}{J}x_1 + \Delta\dfrac{I}{J}x_3 + \Delta\dfrac{u}{J} + \dfrac{v}{J} \\ a(x) = \dfrac{K}{J}x_1 - \dfrac{I}{J}x_3 \\ b(x) = \dfrac{1}{J} \end{cases} \tag{10}$$

Based on the general design in (5) we obtain the control input as follows:

Step 1: Select the DO

We can implement the order 2 or 3 DO and the order 3 is better. So that, we obtain result:

$$d_2: \begin{cases} \hat{d}_2 = p_{21} + l_{21}x_2 \\ \dot{p}_{21} = -l_{21}(x_3 + \hat{d}_2) + \hat{\dot{d}}_2 \\ \hat{\dot{d}}_2 = p_{22} + l_{22}x_2 \\ \dot{p}_{22} = -l_{22}(x_3 + \hat{d}_2) + \hat{\ddot{d}}_2 \\ \hat{\ddot{d}}_2 = p_{23} + l_{23}x_2 \\ \dot{p}_{22} = -l_{23}(x_3 + \hat{d}_2) \end{cases} \tag{11}$$

$$\text{and } d_4: \begin{cases} \hat{d}_4 = p_{41} + l_{41}x_4 \\ \dot{p}_{41} = -l_{41}(a(x) + b(x)u + \hat{d}_4) + \hat{\dot{d}}_4 \\ \hat{\dot{d}}_4 = p_{42} + l_{42}x_4 \\ \dot{p}_{42} = -l_{42}(a(x) + b(x)u + \hat{d}_4) + \hat{\ddot{d}}_4 \\ \hat{\ddot{d}}_4 = p_{43} + l_{43}x_4 \\ \dot{p}_{23} = -l_{23}(a(x) + b(x)u + \hat{d}_4) \end{cases} \tag{12}$$

Step 2: Sliding Surface and control input

The sliding surface and modified sliding surface are selected as follows [2]:

$$\sigma = c_1x_1 + c_2x_2 + c_3x_3 + x_4 + c_3\hat{d}_2 + \hat{\dot{d}}_2 \tag{13}$$

$$\sigma^* = c_1x_1 + c_2x_2 + c_3x_3 + x_4 + c_3\hat{d}_2 + \hat{\dot{d}}_2 - \sigma(0)e^{-\alpha t} \tag{14}$$

The control input is obtained:

$$u = -\frac{1}{b(x)}\Big[a(x) + \hat{d}_4 + \alpha\sigma(0)e^{-\alpha t} + c_1(x_2 + \hat{d}_1) + c_2(x_3 + \hat{d}_2) + c_3(x_4 + \hat{d}_3) + k_l\sigma^* + k_s sat(\sigma^*)\Big] \tag{15}$$

We have $\quad |\sigma^*| \le \lambda_2$ (16)

with: $\lambda_2 = \dfrac{\lambda_1(c_2 + c_3l_{21} + c_4l_{22} + 2c_3) + 2c_3\mu - k_s}{k_l}$ (17)

We absolutely obtain arbitrary small attraction region by using the suitable parameters.

Step 3: Algorithm

a. Find the parameters of (1)
b. Estimate the disturbance of input
c. Selecting $l_{21}, l_{22}, l_{23}, l_{41}, l_{42}, l_{43} > 0$ satisfy matrix $\begin{bmatrix} -l_{21} & 1 & 0 \\ -l_{22} & 0 & 1 \\ -l_{23} & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} -l_{41} & 1 & 0 \\ -l_{42} & 0 & 1 \\ -l_{43} & 0 & 0 \end{bmatrix}$ have eigenvalue belonging the left side of complex coordinate plane. Selecting $\{s_1, s_2, s_3\}$ are positive and big enough. From the equations:
$(s+s_1)(s+s_2)(s+s_3)$
$= s^3 + l_{21}s^2 + l_{22}s + l_{23}$
$= s^3 + l_{41}s^2 + l_{42}s + l_{43}$
We have $\{l_{21}, l_{22}, l_{23}, l_{41}, l_{42}, l_{43}\}$
d. Selecting $\{s_4, s_5, s_6\}$ are positive and big enough. From the equation:
$(s+s_4)(s+s_5)(s+s_6) = s^3 + c_3s^2 + c_2s + c_1$
we obtain $\{c_1, c_2, c_3\}$
e. Selecting $|\Delta| < 0.1, k_s > 0$, $\alpha > 0$ big enough and $k_l > 0$ is bigger than l_{ij}, c_i
f. Finally, we obtain the control law.

4. THE INFLUENCE OF THE PARAMETERS ON THE RESULTS

In this section, we will consider the dependence on the parameters on the results. Matlab simulation results will illustrate these comments.

Simulation results based on parameters (Table 1) as follows:

I	J	M	g	L	K
Inertia moment of joint	Inertia moment of motor	Mass of Link	Gravity	Length of Link	Stiffness
$kg.m^2$	$kg.m^2$	kg	$m.s^{-2}$	m	N.m/rad
1	2	1	10	2	100

4.1 Change the Parameter l_{ij}

When eigenvalues of D_i matrix change, the estimation error in d_2 and d_4 are reduced. Because in [2], we have equation:

$$\dot{\tilde{e}}_i = D_i \tilde{e}_i + E_i d_i^{(r)} \tag{18}$$

So that, the estimation error in d_2 and d_4 are reduce if the eigenvalues less than 0 and far from imaginary axis.

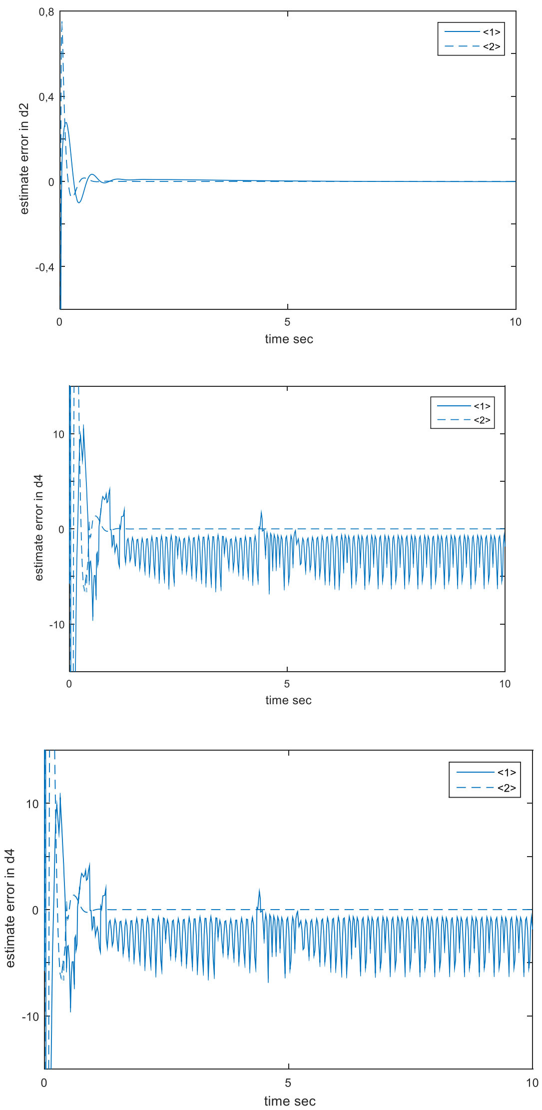


Figure 1. Result of change l_{ij}

$<1>\ l_{21} = l_{41} = 100, l_{22} = l_{42} = 20$

$<2>\ l_{21} = l_{41} = 100, l_{22} = l_{42} = 1875$

4.2 Change of the Parameter c_i

If the set of parameters c makes the polynomial's root less than 0 and far from imaginary axis, the $\tilde{d}_2, \tilde{d}_4, \sigma^*, x_1$ to come to the domain of attraction fast.The domain of attraction is less. Because:

We have : $\exists T < \infty : |\sigma^*| < \lambda_2 \ \ \forall t > T$ (19)

Let's look at the case at the border:

$$c_1 x_1 + c_2 \dot{x}_1 + c_3 \ddot{x}_1 + \dddot{x}_1 = \pm\lambda_2 + \sigma(0)e^{-\alpha t} \tag{20}$$

The root of the equation (20) is:

$$x = \frac{\pm\lambda_2}{c_1} + Ae^{-s_1 t} + Be^{-s_2 t} + Ce^{-s_3 t} + De^{-s_4 t} + Ee^{-\alpha t} \tag{21}$$

So that, in the best case (at the boundary) , the output signal come to the neighborhood of 0 when we select parameters to $\{s_1, s_2, s_3, s_4, \alpha\}$ are big. Then $c_1 = s_1 s_2 s_3 s_4$ is big number.

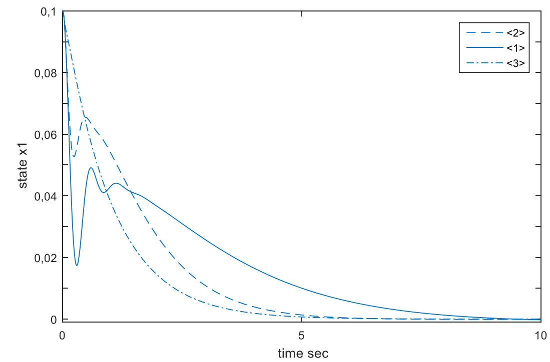


Figure 2. Result of change c_1

$<1>\ c_1 = 125, c_2 = 75, c_3 = 15$

$<2>\ c_1 = 1000, c_2 = 300, c_3 = 30$

$<3>\ c_1 = 1000000, c_2 = 30000, c_3 = 300$

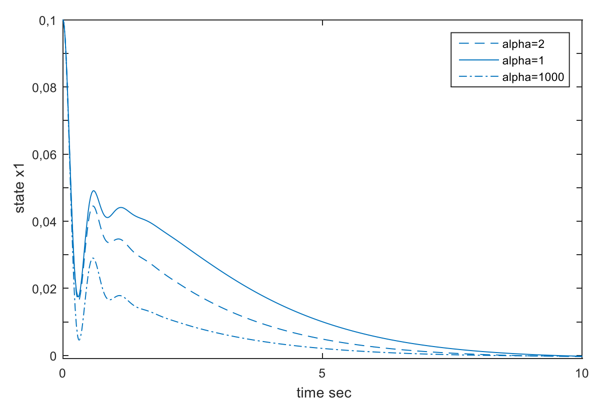
4.3 Change of the Parameter α

If α become large, $\tilde{d}_2, \tilde{d}_4, \sigma^*, x_1$ will fast come to the attractive region.

We select $\sigma^* = \sigma - \sigma(0)e^{-\alpha t}$ to have $\sigma^*(0) = 0 \le \lambda_2$.

The component $e^{\alpha t}$ is selected because $\forall n$: $\lim\limits_{t\to\infty} \frac{d^n e^{\alpha t}}{dt^n} = 0$.

However, if α become large, $\frac{d\sigma^*}{dt}(0)$ increase and σ^* can be cone large when $t \approx 0$.



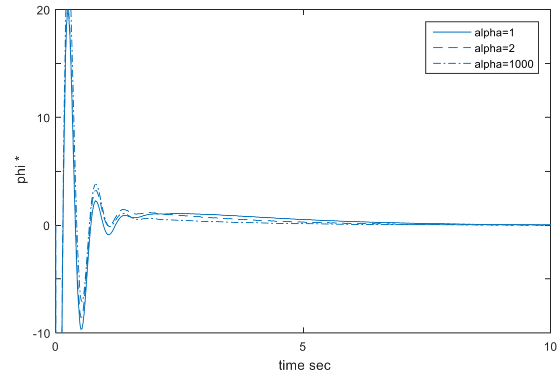


Figure 3. Result of change α

4.4 Change of k_l and k_s

Because of equation (17) , we think that if k_l and k_s increase , σ^*, x_1 will come to the attractive domain fast. The domain of attraction is less.

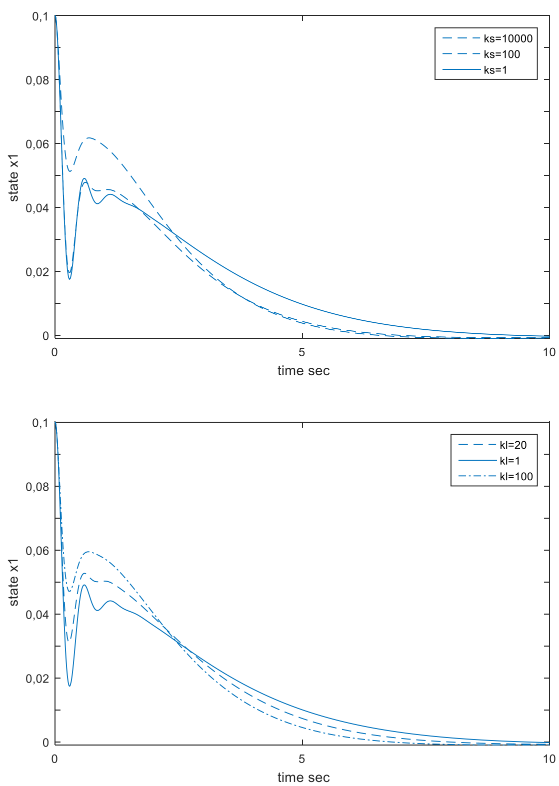


Figure 4. Result of change of k_l and k_s

4.5 The degree of the DO

If we increase the degree of the DO, we will have the better results. The estimation error in d_2 and d_4 are reduced.

We can find the different between 2nd-DO and 3rd-DO from Figure5.

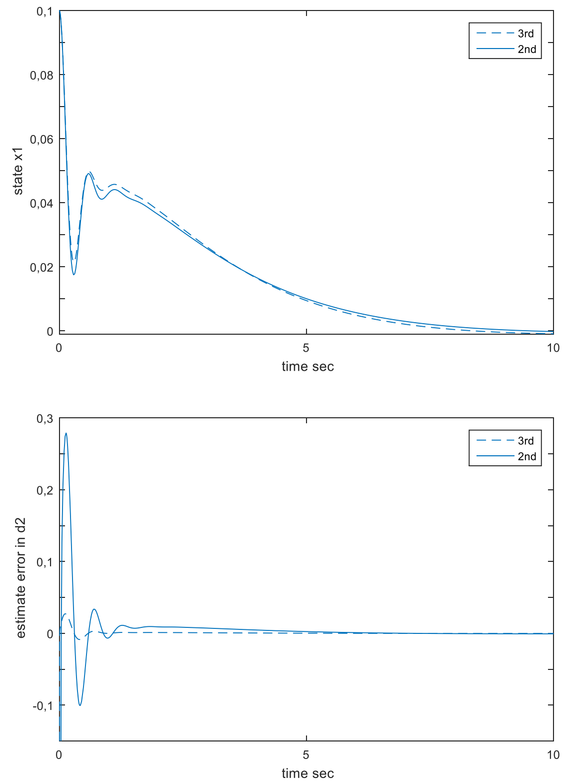


Figure 5. Comparison between 2nd-DO and 3rd-DO

5. CONCLUSION

This work proposed the sliding mode control law based on DO for single link manipulator. Moreover, we absolutely obtain arbitrary small attraction region by using the suitable parameters. Simulation results demonstrate the proposed controller.

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